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The Gravitational Constant G

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I. Introduction

The gravitational interaction between any two particles of matter is stated in Newton's law of gravitation

$$F = G\frac{m_1 m_2}{d^2}$$

where F is the force of attraction between any two particles of matter in the universe having masses of m_1 and m_2, d is the distance between the particles, and G is the constant of proportionality. Gravitational interaction possesses two rather unique properties: great universality and extreme weakness. Both of these features contribute to the difficulty of measuring the gravitational constant, G.

The universality of G is evident from the fact that every particle of mass is coupled directly to every other particle of mass in the universe by a force of attraction, and from the fact that careful experiments have failed to show any variation of G with such things as the nature and magnitude of the attracting masses, their states of chemical combination, their temperatures, or the amount of matter placed between the attracting masses (1,2,3). The extreme weakness of the gravitational interaction can be illustrated by the fact that the gravitational attraction between two spheres each with a mass of 10 kg and their centers separated by 0.15 m is approximately 3×10^{-11} newtons or about 3×10^{-13} times the force of gravity on each sphere. Thus the experimenter is somewhat restricted in available techniques, and it is not possible to isolate or shield the test masses from the gravitational fields and gradients produced by all of the other masses in the universe.

Considering the great precision of the astronomical measurements of the paths of celestial bodies one might think that somehow G could be found from these data. However, it turns out that the product GM, where M is the mass of the body, is obtained from such astronomical observations instead of G or M. Consequently it is necessary to have an independent determination of G if the mass M and the mean density of the earth or other celestial bodies are to be found. Experimental methods of measuring G have fallen into three general classes: 1) comparison of the earth's pull on a body with that of the attraction of a large natural mass such as a mountain or other topographical features; 2) comparison of the earth's pull on a body with that of a known mass as in the common balance experiments; 3) measurement of the force between known masses in the laboratory. Up to the present time only 2) and 3) of the above classification have given reliable values of G.

Table I gives a partial list of some values reported.

It is difficult to estimate the precision of the values obtained before the work of Poynting (1891) and of Boys (1895). However, an examination of the results of the last half dozen workers listed in Table I shows that the variation of the values obtained for G is substantially larger than what might be expected from the precision of the data indicated in the papers. Recent analyses (4,5) of the data of Heyl and his associates which is usually considered to be the most accurate of that listed in Table I give $(6.670 \pm 0.015) \times 10^{-11} Nm^2kg^{-2}$ for G.

In 1965 the authors proposed[6] a new experimental method for the determination of G, which showed promise of much improved precision. Briefly the principle of the method is illustrated in Fig.1.

Two large spherical masses (tungsten spheres) are mounted on a rotary table which can be driven about its axis of rotation by a specially designed electric motor. Also mounted from the same rotary table is an airtight chamber in which a small horizontal cylinder made of copper is suspended by means of a quartz torsion fiber fastened to the top of the cylinder and accurately hanging in the axis of rotation. This small horizontal cylinder is commonly

TABLE I

Name	Year	Method	$G(10^{-11} Nm^2kg^{-2})$
Cavendish	1798	Torsion-balance(deflection)	6.754
Reich	1838	Torsion-balance(deflection)	6.61
Baily	1842	Torsion-balance(deflection)	6.475
Von Jolly	1881	Common balance	6.465
Wilsing	1889	Metronome balance	6.596
Poynting	1891	Common balance	6.698
Boys	1895	Torsion-balance(deflection)	6.6576
Braun	1896	Torsion-balance(oscillation)	6.6579
Eötvös	1896	Torsion-balance(oscillation)	6.65
Richarz	1898	Common balance	6.685
Burgess	1901	Torsion-balance(deflection)	6.64
Heyl	1930	Torsion-balance(oscillation)	6.670 ± 0.005
Zahradnicek	1932	Torsion-balance(resonance)	6.659 ± 0.02
Heyl and Chrzanowski	1942	Torsion-balance(oscillation)	6.673 ± 0.003

called the small mass system as contrasted to the large spheres which are referred to as the large mass system.

The gravitational interaction between the two mass systems tends to align the longitudinal axis of the small mass cylinder with the centers of the two large spheres, changing the angle, θ. However, the angle θ never changes appreciably because a beam of light from a source mounted on the rotary table is reflected from a mirror mounted on the axis of the quartz fiber and falls on a photocell, also mounted on the rotary table. The photocell senses the minute changes in θ, and produces an "error" signal which is used to drive the motor which rotates the table, thus maintaining θ constant. With the angular separation, θ, remaining constant, the small mass system experiences a constant torque, which in turn causes a constant angular acceleration of the rotary table. This acceleration can be determined very accurately by measuring the period of the rotating table, and can be shown to be a direct measure of G.

This method possesses three novel features which contribute to the potential for improved accuracy. First, the interaction force of the two masses is manifested in an acceleration (change of frequency) rather than in a deflection. The effect of the interaction is cumulative and can be integrated over a long period of time, thus improving precision (the noise level is also automatically reduced). Second, the two mass systems rotate about an axis many times during a measurement, and hence the effects of gravitational fields or field gradients caused by extraneous masses are effectively cancelled except for higher order effects. Third, the coordinates or positions of the interacting masses with respect to a rotating coordinate system do not change during a measurement and hence their distances can be accurately determined.

2. The Determination of G

A theoretical analysis(6) of the gravitational torque system gives:

$$G = \frac{\alpha \pm \dot{\omega}_{w/o}}{A(1 + B + C \ldots)} \tag{1}$$

where

$$A = \frac{3M}{R^3} \frac{\left[\frac{1}{3} - \frac{1}{4}\left(\frac{2a}{L}\right)^2\right]\sin 2\theta}{\left[\frac{1}{3} + \frac{1}{4}\left(\frac{2a}{L}\right)^2 + \left(\frac{2}{L}\right)^2 \frac{I_s}{m}\right]}, \tag{2}$$

$$B = \frac{5}{6}\left(\frac{L}{2R}\right)^2 \frac{\left[\frac{1}{5} - \frac{1}{2}\left(\frac{2a}{L}\right)^2 + \frac{1}{8}\left(\frac{2a}{L}\right)^4\right]}{\left[\frac{1}{3} - \frac{1}{4}\left(\frac{2a}{L}\right)^2\right]}(7\cos^2\theta - 3), \tag{3}$$

$$C = \frac{1}{48}\left(\frac{L}{2R}\right)^4 \frac{\left[\frac{1}{7} - \frac{3}{4}\left(\frac{2a}{L}\right)^2 + \frac{5}{8}\left(\frac{2a}{L}\right)^4 - \frac{5}{64}\left(\frac{2a}{L}\right)^6\right]}{\left[\frac{1}{3} - \frac{1}{4}\left(\frac{2a}{L}\right)^2\right]} \quad (4)$$

$$\times[1386\cos^4\theta - 1260\cos^2\theta + 210],$$

and

G	=	The Newton Gravitational Constant,
α	=	Measured angular acceleration with spheres on the table,
$\dot{\omega}_{w/o}$	=	Angular acceleration with spheres removed,
M	=	Mass of one sphere,
R	=	Distance from axis of rotation to center of mass of the spheres,
m	=	Mass of small mass system cylinder,
a	=	Radius of small mass system cylinder,
L	=	Length of small mass system cylinder,
I_s	=	Moment of inertia of small mass system stem,
θ	=	Angle between the longitudinal axis of the small mass system cylinder and the imaginary line of centers of the spheres.

The three terms in the denominator of Eq. 1 correspond to the first three non-zero terms resulting from an expansion of the gravitational potential of the small mass system cylinder in terms of Legendre polynomials. Additional terms can be calculated if necessary, but they do not contribute to the present level of precision. It is important to note the R^3 dependence in Eq. 1.

The experimental task is to determine $\alpha, \dot{\omega}_{w/o}, R, M, a, L, I_s, m$, and θ as accurately as possible. Five of these (a, L, I_s, M, and m) can be considered as constants of the apparatus and can be measured directly, independently of the experimental observations. The other four ($\alpha, \dot{\omega}_{w/o}, R$, and θ) must be determined as part of the observation. A typical set of values, including estimated errors, is given in Table II.

TABLE II

M	=	$10.49012 \pm .00007$ kg
R	=	$7.6178 \pm .0005$ cm
a	=	$.1985 \pm .0003$ cm
L	=	$3.8105 \pm .0004$ cm
m	=	$4.051205 \pm .000005$ gm
I_s	=	$.0445103$ gm cm^2
θ	=	$.7835 \pm .0002$ rad
α	=	$(4.3299 \pm .0022)\times10^{-6}$ rad/s^2
$\dot{\omega}_{w/o}$	=	$(-\ 0.3431 \pm .0022)\times10^{-6}$ rad/s^2

The procedure in carrying out this experiment consists of first assembling the small mass system in the gas tight chamber and properly positioning it. The quartz fiber is adjusted to coincide with the axis of rotation and the tracking optical lever is set so that the effective twist in the fiber is as close to zero as possible. The chamber is evacuated, filled with helium and sealed. The tungsten spheres are placed in their mountings and are positioned equidistance from the axis of rotation in the same horizontal plane as the axis of the cylinder of the small mass system and with the centers of mass of the two spheres on a line through and perpendicular to the axis of rotation. The entire apparatus, located in a small room, is then allowed to come to temperature equilibrium with the servo system operating and the rotary table stationary. This usually requires one day or more. The temperature of the room is controlled to a point just above the apparatus in a manner which produces a slight positive vertical gradient.

After the apparatus has reached temperature equilibrium the absolute value of R is determined by comparison with a precision gauge block using a pair of white light interferometers in conjunction with an electronic indicator. In this manner R is determined to within $\pm 5 \times 10^{-5}$ cm. The apparatus is then allowed to stand until late at night when there are few people in the building and the community is relatively quiet. The observations are then begun by releasing the rotary table which is supported on a gas bearing and locking the optical sensor and servo system into a high gain mode of operation for tracking the small mass system. The rotary table and mass systems are then allowed to accelerate for 10 to 20 revolutions with each revolution being accurately timed to determine α. Next with the servo system returned to a low gain mode the spheres are carefully removed, usually without changing the speed of the rotary table appreciably. The servo is then returned to the high gain mode and the table is tracked for another 10 to 20 revolutions to determine $\dot{\omega}_{W/O}$. The procedure of placing the spheres in the mounts and removing them is repeated several times without stopping the rotary table. Finally the rotary table is stopped and R is again measured. These measurements have shown that during the process of removing and replacing the spheres on the table R does not change more than 5×10^{-4} cm. Greater precision requires stopping the observations and remeasuring R each time the spheres are replaced on the table.

The acceleration measurements shown in Fig. 2 are typical of very recent observations. The same data are displayed in Fig. 3 with the acceleration scale magnified by a factor of 50. The first few revolutions after removing or replacing the spheres are neglected because of large vibrations of the small mass system. Even with the greatest of care, placing the spheres on the table results in some slight mechanical shock and several revolutions are required before the vibrations damp out. As would be expected, the small mass system is very sensitive to vibration.

Seven sets of observations similar to Fig. 3 have been obtained since July 22, 1971 and in all cases α and $\dot{\omega}_{w/o}$ show a dependence on angular velocity. Possible sources of this dependence being presently investigated include magnetic fields fixed in the laboratory, variations in the temperature of the apparatus, and the circulation and acceleration of the gas in the chamber containing the small mass system. This velocity dependence was not noted in our earlier measurements but may have been masked by the noise in the acceleration measurements at that time. In any event by proper treatment of the data it is now possible to determine a value for $\dot{\omega}$ with a statistical uncertainty for a single set of observations of 1 part in 3000.

Recent observations are summarized in Table III. We have not had time to perform a careful analysis of these data or to completely check for systematic errors. The values of G shown in Table III were computed from Eq. 1 to normalize the data for comparisons and are not presented as absolute determinations of G for obvious reasons. None of the observations was made with magnetic shielding around the small mass chamber. Each run represents some change in operating conditions, either planned or accidental. For example run 7/22/71 represents a larger value of R. The table was rotating in the opposite direction during run 8/17/71. Higher temperatures and poor temperature stability existed during run 8/19/71 because of a failure of the building air conditioning system. There was a very large twist in the fiber during run 8/21/71, hence a large $\dot{\omega}_{w/o}$. A large number of observations of this type will need to be made before the best operating conditions can be determined and, hopefully, all non-gravitational interactions with the small mass system identified and properly accounted for.

3. Summary

The most attractive feature of the present apparatus is its high sensitivity for measuring very small forces (torques). It is expected that the sensitivity can be improved further by replacing the quartz fiber with a magnetic suspension. The major known

TABLE III

(M, a, L, m, θ, and I_s same as given in Table II)

Date	$\dot{\omega}$ 10^{-6} rad/s^2	R cm	G 10^{-11} m^{2}kg^{-2}
7/22/71	4.4388	7.768 ± .003	6.6942
8/13/71	4.7022	7.61798	6.6857
8/16/71	4.6731	7.61819	6.6448
8/17/71	4.6738	7.61820	6.6459
8/19/71	4.6304	7.61813	6.5840
8/20/71	4.6941	7.61815	6.6746
8/21/71	4.7071	7.61827	6.6934

(R values for 8/13/71 through 8/21/71: ± .00005) difficulties associated with this method of determining G are:

1. The effect of the gas in the small mass system chamber. Eventually the experiment will have to be done in a vacuum, as originally proposed.

2. The effect of magnetic fields. It was intended that the small mass system be made from high purity oxygen free (diamagnetic) copper but unfortunately the present cylinder was found to be paramagnetic. A new small mass system is being constructed from pure oxygen free copper and magnetic shielding of the small mass system is planned. Other materials are being considered for the small mass system.

3. Temperature stability. This has been greatly improved but may again become a problem when greater precision is attempted.

4. Vibrations. Because of the extreme sensitivity of the small mass system to vibrations the present experiments were performed with the small mass chamber filled with gas. Other methods of damping the small mass system are being considered. The coupling of torque to the small mass system through vibration is unknown. Fortunately the errors in the determination of acceleration resulting from vibration induced variations in timing can be reduced to an arbitrarily small value by averaging over an adequate number of revolutions of the rotary table.

The value we previously reported (6), $G = (6.674 \pm 0.012) \times 10^{-11} Nm^2/kg^2$, will remain our best until enough observations can be accumulated to permit the gravitational forces to be reliably isolated.

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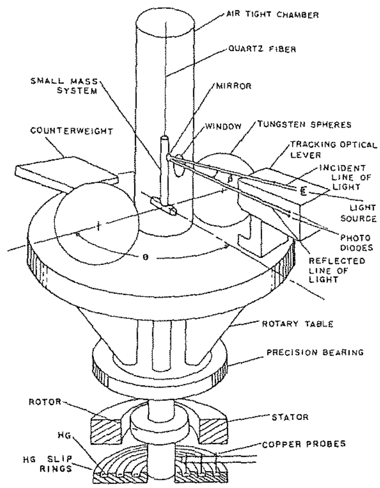

Fig. 1. Schematic Drawing of Experimental Apparatus.

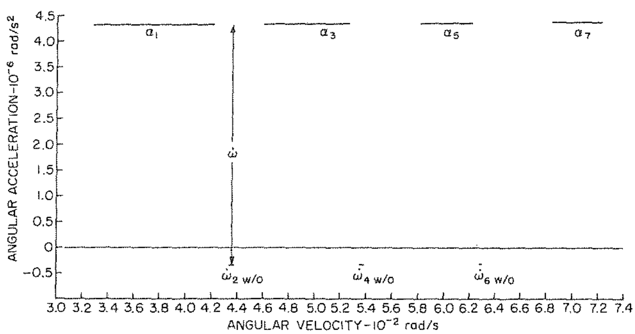

Fig. 2. Measurements of Angular Accelerations α, $\dot{\omega}_{w/o}$ - August 16, 1971.

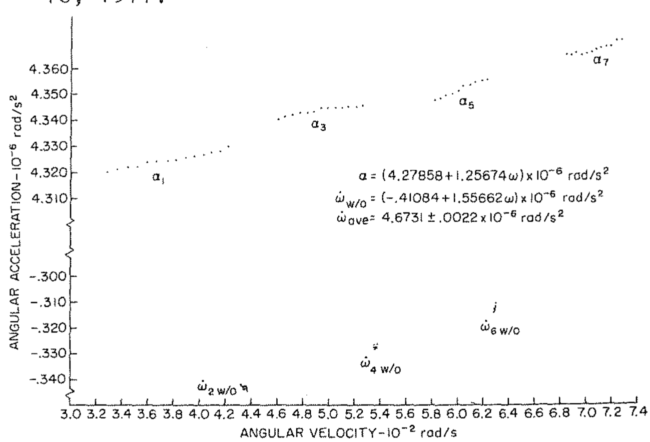

Fig. 3. Measurements of Angular Accelerations α, $\dot{\omega}_{w/o}$ - August 16, 1971.